INVESTIGATING $\Delta m = \pm 1$ TRANSITIONS IN A CAESIUM FOUNTAIN CLOCK – CHALLENGES IN PRECISION MEASUREMENTS OF THE *g*-FACTOR RATIO

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ABSTRACT

Theoretical calculations of atomic structure are constantly being refined and are now reaching a degree of accuracy where the final discrepancies to tabulated measurements (see e.g. [1]) might be due to the experimental values [2,3]. This uncertainty also exists for caesium, where the difference in the measured nuclear magnetic moments for free caesium atoms [4], free caesium ions [5] and ions in solution [6] do not agree with calculated shielding factors [7].

The best known ratio of the nuclear and electronic *g*-factors in free caesium atoms is given in [4] as $g_I / g_J = -1.9917400(26) \cdot 10^{-4}$. We aim to remeasure this ratio by investigating a set of hyperfine transition frequencies using the two available caesium fountains CSF1 and CSF2 at PTB. In this report we describe the results of the first tests and the challenges they show for a precision measurement.

PRINCIPLE OF MEASUREMENT

Fountain clocks are designed to probe the hyperfine transitions in the electronic ground state of the caesium atom. The relevant energy levels in the fountain's magnetic quantization field B_c are described by the Breit-Rabi equation, which takes on the form

$$\left.\begin{aligned} E_{|4,m_4\rangle} &= -\frac{hf_{\mathrm{HFS}}}{16} + g_I\, m_4\, \mu_{\mathrm{B}} B_c + \frac{hf_{\mathrm{HFS}}}{2}\sqrt{1+\frac{m_4\, x}{2}+x^2} \\ E_{|3,m_3\rangle} &= -\frac{hf_{\mathrm{HFS}}}{16} + g_I\, m_3\, \mu_{\mathrm{B}} B_c - \frac{hf_{\mathrm{HFS}}}{2}\sqrt{1+\frac{m_3\, x}{2}+x^2} \end{aligned}\right\} \quad \text{with } x = \frac{(g_J - g_I)\mu_{\mathrm{B}} B_c}{hf_{\mathrm{HFS}}} \qquad (1)$$

for the $F = 4$ and $F = 3$ states with their magnetic sublevels m_4 and m_3. In this equation g_J and g_I are the electronic and nuclear *g*-factors under investigation and $f_{\mathrm{HFS}} = 9.192631770$ GHz is the caesium hyperfine transition frequency. By comparing the transition frequencies

$$f_{(m_3,m_4)} = \frac{E_{|4,m_4\rangle} - E_{|3,m_3\rangle}}{h} \qquad (2)$$

for different $\Delta m = m_4 - m_3$, it is possible to separate the effects of electronic and nuclear magnetic moments and measure the ratio of g_I to g_J even without precise knowledge of the effective field B_c.

We have chosen a measurement scheme involving the transitions $|F = 3, m_3 = 0\rangle$ to $|F = 4, m_4 = 0\rangle$, $|F = 3, m_3 = -1\rangle$ to $|F = 4, m_4 = 0\rangle$ and $|F = 3, m_3 = -1\rangle$ to $|F = 4, m_4 = -1\rangle$ (referred to as [0 to 0], [-1 to 0] and [-1 to -1] in the following). The [0 to 0] transition is the clock transition that the fountain clocks are normally operated on, and its frequency is provided by the fountain CSF1 during the measurement. The second fountain CSF2 operates with its state selection set to prepare atoms in the $|F = 3, m_3 = -1\rangle$ state and alternates between measuring the [-1 to -1] and [-1 to 0] frequencies.

A linearized model gives the *g*-factor ratio based on this set of transition frequencies as

$$\frac{g_I}{g_J} = \frac{2f^*_{(-1,0)} - f^*_{(-1,-1)}}{2f^*_{(-1,0)} - 9f^*_{(-1,-1)}} \qquad \text{with} \quad f^*_{(m_3,m_4)} = f_{(m_3,m_4)} - f_{(0,0)} \quad . \qquad (3)$$

The simplicity of this equation makes it useful for quick evaluations and its accuracy is better than 10^{-10} for the weak magnetic quantization field of $B_C = 146$ nT employed in the fountain, providing no limitation at the current state of the experiments. When it becomes necessary, a calculation based on the full Breit-Rabi formula can be used instead.

A direct measurement of g_I is not feasible in the fountain as it would require accurate knowledge of the magnetic fields experienced by the atoms on their trajectory. But since the value of g_J is known to great precision, a measurement of the ratio would immediately lead to better value for g_I.

When operated as a clock, CSF1 and CSF2 reach relative uncertainties of $0.8 \cdot 10^{-15}$ for the measurement of the [0 to 0] transition [8]. If the same accuracy could also be reached for the other two transitions, the resulting absolute uncertainty of the g-factor ratio would be $2 \cdot 10^{-9}$, approximately an order of magnitude worse than stated in [4]. The uncertainty can be reduced by increasing the magnetic quantization field.

FIRST TEST

A first test measurement was recently performed to test the viability of this scheme. The fountain CSF2 was first locked to the [-1 to -1] transition for several hours, then to [-1 to 0] and finally again to [-1 to -1]. Preliminary corrections were applied to the frequency measurements using the values found for normal clock operation. The measurements were then compared to the corrected [0 to 0] transition frequency simultaneously measured by CSF1. As the quantization field varies over time, the measurements show a considerable, mostly linear drift of the transition frequency over the chosen period. To take this into account, a linear fit was applied to the stretches of [-1 to -1] data as seen in Fig. 1(a). This interpolation and the literature values of the g-factors [4] were then used in an equation derived from (3) to predict the relative [-1 to 0] frequency. As seen in (1), $E_{|4,\ m4=0>}$ is much less sensitive to changes of the magnetic field, and the expected frequency drift is therefore almost exactly half of that observed for the [-1 to -1] transition.

The reduced drift can be seen in Fig. 1(b), but there is also a clear discrepancy of around 24 mHz (corresponding to $2.6 \cdot 10^{-12}$ of the clock transition frequency) between the prediction and the measurements, well beyond the noise of the frequency measurements. While the prediction is limited in its accuracy by the use of the linearized model, it should still provide frequencies accurate to better than 1 mHz, clearly indicating the presence of a systematic shift.

Using the measured frequency values for the [-1 to 0] transition and the corresponding interpolation for [-1 to -1] in (3) gives a series of values for the g-factor ratio. The averaged value is $g_I / g_J = -2.0489 \cdot 10^{-4}$ and differs from the literature value by about 3%. The corresponding absolute difference of $-5.7 \cdot 10^{-6}$ is much larger than the standard deviation of the measurements, which is $6.4 \cdot 10^{-7}$. Assuming the ideal case of normally distributed, random noise superimposed on a stable systematic offset, the statistical uncertainty reached within 8 hours is $5.4 \cdot 10^{-8}$.

CHALLENGES FOR A PRECISION MEASUREMENT

The primary limitation both on systematic and statistical uncertainty of the measurement is due to the nature of the [-1 to 0] transition when it is excited by the fountain's microwave cavity that was designed for using the [0 to 0] transition. The components of the RF magnetic field in the cavity that drive $\Delta m = \pm 1$ transitions need to be orthogonal

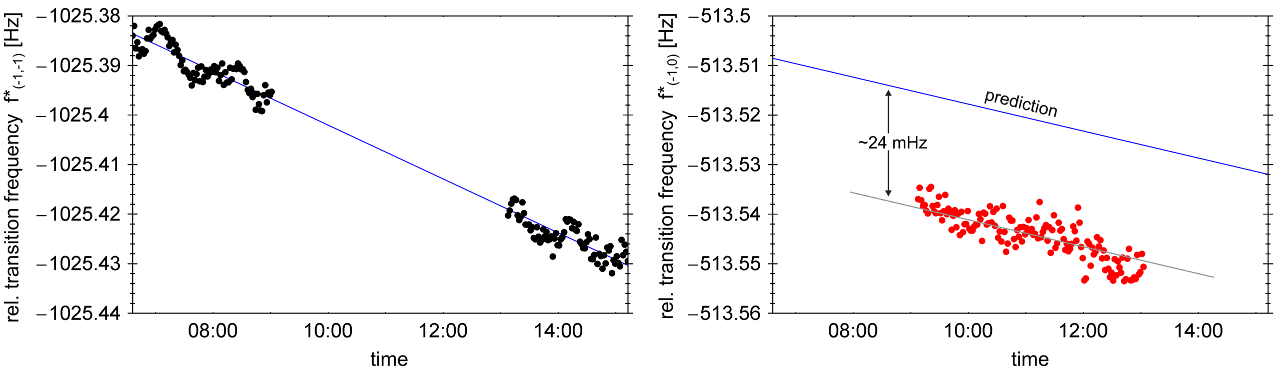

Fig. 1. (a, left) : Measured frequencies (each point marks a 100 s interval) of the [-1 to -1] transition (black) together with linear fit (blue line)

(b, right) : Measured frequencies of the [-1 to 0] transition (red) and prediction (blue line) from the fit of [-1 to -1]. The gray line corresponds to a constant frequency offset close to 24 mHz.

to the vertical quantization field and therefore lie in the horizontal plane. As the horizontal component is present only due to the curvature of the -mostly vertical- field lines of the TE011 mode (see Fig. 2), it varies strongly with radial position and changes sign between the top and the bottom of the cavity. When the microwave is resonant with the $\Delta m = \pm 1$ transition, this causes the second half of the cavity passage to undo the Bloch vector rotation acquired in the first half. The resulting transition probability P is then zero both for a single cavity passage and the complete Ramsey cycle of the fountain, as seen in Fig. 3. Away from resonance the cancellation of Bloch vector rotation gets less complete, leading to a “split fringe” shape in the expected spectrum.

The shape of the spectrum is much more sensitive to the details of the magnetic quantization field than for the clock transition. This was found to cause a residual fringe amplitude of approximately 1% near the [-1 to 0] transition frequency that made it possible to perform the test measurement in the way described here. However, this amplitude is still quite small, leading to a degraded signal-to-noise ratio and causing the increased short-term instability of the [-1 to 0] measurement visible in Fig. 1.

The small amplitude also causes any residual asymmetries to have strong pulling effects. An estimate based on the local slope of the envelope center compared to the slope of the sides of the central fringe predicts a frequency shift of roughly -15 mHz for the spectrum shown, which is at least a significant contribution to the observed discrepancy from the expected result.

There is another effect of using the horizontal components of the RF magnetic field that might be of similar magnitude: The azimuthal angle of the horizontal field component determines the effective phase of the superposition state created by the first cavity passage. This has no effect if the second passage occurs at the same azimuthal angle, but any change in angle will directly translate into a phase shift of the Ramsey fringe. The fountain adjustment for a vertical launch direction ensures that these angle changes are small and mostly average out over the different trajectories in the expanding cloud, but any residual shift has a large effect on the measured g-factor ratio.

Further systematic shifts of somewhat smaller magnitude could be induced by blackbody-radiation or cold collisions. The frequency measurements of both fountains were corrected for the shifts found in normal operation, but the required corrections are expected to be different when measuring the [-1 to 0] and [-1 to -1] transitions and require careful investigation.

If all these systematics can be overcome, the drifts and fluctuations in the strength of the quantization field are a minor problem, as they mostly affect the short term stability of the measurement: The magnetic field does not need to be constant as long as the error on the interpolated [-1 to -1] transition frequency is small. Since the field fluctuations occur on timescales of 24 hours for larger changes, with a superimposed oscillation of approximately 50 minutes period due to the air-conditioning, this can be achieved by alternating between [-1 to -1] and [-1 to 0] measurements more often, possibly every few shots.

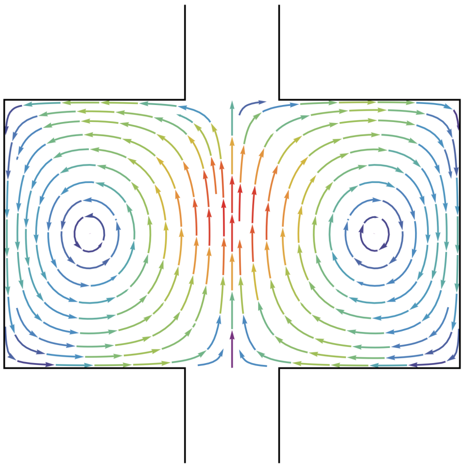

Fig. 2. Cross-section of the cylindrical cavity showing the RF magnetic field lines for the TE011 mode

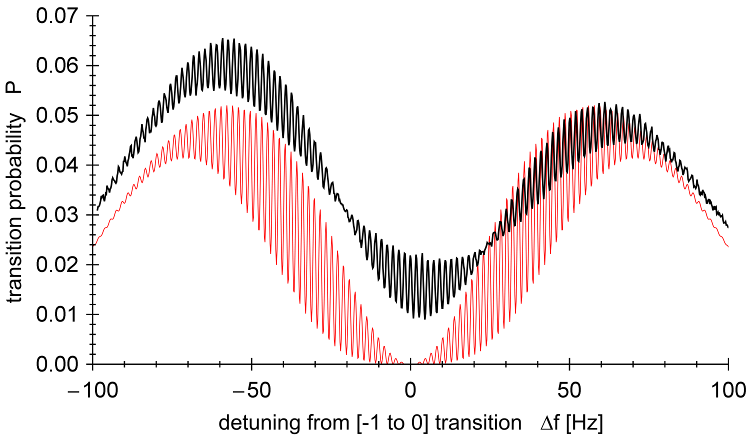

Fig. 3. Predicted (red) and measured (black) transition probability spectrum for the [-1 to 0] transition

PLANNED IMPROVEMENTS

Two improvements are currently planned for future measurements:

By inverting the phase of the microwave signal when the atoms reach the half-way point of the cavity, it is possible to prevent the cancellation of Bloch vector rotation that usually occurs due to the reversal of the horizontal field component during the cavity passage. Preliminary tests have already demonstrated this and a new version of the microwave synthesis currently under development will provide the option of a well-controlled 180° phase switch using direct digital synthesis (DDS) techniques. The increased fringe amplitude will improve signal-to-noise ratio and short-term instability as well as reducing frequency pulling due to asymmetries.

The frequency error resulting from a change in azimuthal angle between cavity passages is constant as long as the atomic trajectories remain unchanged. It can therefore be removed from the resulting value of the *g*-factor ratio by measuring the transition frequencies at different strengths of the magnetic quantization field.

SUMMARY

Operating the caesium fountain on the $\Delta m = \pm 1$ transitions causes effects to become relevant that are very well suppressed in normal clock operation. Careful tuning of the system, and in particular of the quantization field both in the cavity and in the drift region, is required to limit frequency errors introduced by the asymmetry of the spectrum for the magnetically sensitive transitions.

Even under optimum conditions it is unlikely that a measurement of the *g*-factor ratio in our fountains can reach the accuracy claimed by previous measurements, but it could provide an independent remeasurement in a regime of magnetic field strengths below 1 µT, much less than the 1.5 mT used in [4]. It might also help resolve the discrepancy between theoretical and observed nuclear shielding factors [7] or provide yet another value as the atoms in the fountain find themselves in an ensemble of ultracold atoms rather than in a thermal cloud. Given that a dependence of g_J on buffer gas pressure has already been observed [9], this might not be altogether surprising.

At the very least, investigating the shape of the $\Delta m = \pm 1$ transitions will provide more insight into the processes occurring during the Ramsey cycle and could lead to improved diagnostics of magnetic field problems that might affect operation as a primary frequency standard.

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